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The impact of profit-sharing investment accounts on shareholders' wealth ☆

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ABSTRACT

This study evaluates the impact of profit-sharing investment accounts (PSIAs) on shareholders' wealth. Using probabilistic tools, we derive an explicit formula for PSIA value. We apply this formula to calculate account values for a sample of 52 banks in 13 countries and determine an empirical rank order of value-drivers. We also assess the impact of returns' smoothing schemes on shareholders' wealth, namely the use of reserves and/or subsidies to adjust cash returns to investment account holders. We find that the value of PSIAs varies widely amongst banks, and that originating high-quality assets dominates returns' smoothing in the value creation process. We also find that if subsidies are used without reserves to smooth returns, then shareholders' wealth is destroyed in the long term for 77% of banks in our sample. This finding supports the practice of prioritising reserves over subsidies to smooth returns.

1. Introduction

Islamic banks source a majority of their retail funds using Profit-Sharing Investment Accounts (PSIAs). According to Sundararajan (2011), 60% of Islamic bank funding is from PSIAs. The nearest perceived equivalent to investment accounts are interest-bearing time deposits (Archer and Karim, 2009). However, PSIAs are not bank liabilities, although they are sometimes presented as such in the financial statements of Islamic banks. Instead, PSIAs are equity investments in which investment account holders' (IAHs) capital is invested in return-bearing assets managed (and originated) by the Islamic bank. In return for its funds' management role, the bank shares ex-ante uncertain profits with account holders, and this is a major source of revenue for Islamic banks (Archer and Karim, 2006).

The objective of this paper is to determine the value created for shareholders from PSIAs. Whilst conventional deposits have been valued (Hutchison and Pennacchi, 1996; Jarrow and Van Deventer, 1998; Sheehan, 2013), the valuation of PSIAs has not been reported. As a result, it has also not been possible to enumerate the monetary impact on shareholders' wealth of smoothing cash returns paid to PSIAs, and in particular, to say whether this destroys shareholders' wealth if funded by bank equity.

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To clarify, Islamic banks do not pay the contractual share of profits earned in each accrual period over to investment account holders ("investors") automatically. Instead, cash returns to investors may be adjusted ex-post so that banks remain competitive relative to a conventional deposit rate benchmark and thereby avoid account withdrawals (Sundararajan, 2007; Farook et al., 2012; Aysan et al., 2018). Capital applied to smooth returns is sourced from either a reserve funded jointly by shareholders and investors (the Profit Equalisation Reserve, PER), a reserve funded solely by investors (the Investment Risk Reserve, IRR), or by donating part of the profits attributable to shareholders (hereafter "subsidies"). If shareholders' equity is used for returns' smoothing (via PER and/or subsidies), then the benefits of using equity need to outweigh the corresponding opportunity cost, namely the returns generated for shareholders by the investment of bank equity in positive NPV projects. To determine the intrinsic value of PSIAs (i.e. without smoothing), and to investigate the overall impact of PSIAs on shareholders' wealth when returns' smoothing is funded by shareholders' equity, we develop a theoretical model of PSIAs. The model uses a contingent-claims approach, in which PSIA value is an expectation of the bank's discounted share of future profits in respect of an ex-ante uncertain future account volume. There are two state variables. These are a stochastic rate of return on underlying assets, and a stochastic conventional deposit rate. Together with a pre-defined profit-sharing ratio, these state variables define the relative return, i.e. return on investment accounts relative to conventional deposit rates. In turn, the relative return drives changes in investment account volume to capture account retention and switching behaviour.

This paper is a timely addition to the literature, informing both practitioners and academics. Since 2005 there has been a pronounced increase in M&A activity for Islamic banks, some of which has involved conventional banks (see Appendix A – cumulative deal volume and cumulative dollar value of completed mergers and acquisitions for Islamic banks from 1984 to 2019). Total completed mergers and acquisitions (by value) of Islamic banks in the period from 2010 to 2019 was approximately double that of the period 2000–2010. Increased consolidation activity highlights the need for a model of PSIA value for M&A due diligence to determine the fair value of Islamic banks and corresponding purchased goodwill.

Our paper makes several significant contributions. First, we develop for the first time an explicit formula for PSIA value. Second, we quantify PSIA values for a sample of 52 Islamic banks from 13 countries, and rank the importance of PSIA value-drivers to infer how bank policies may be steered to higher value creation for shareholders. Third, we evaluate the long-term impact on shareholders' wealth when returns' smoothing is funded by shareholders' equity.

The results of our analyses are compelling. We find a substantial variation in PSIA values both within each country and across countries. PSIAs range in value from single digits to just over 60% of account size. This wide variation confirms a need to accurately reflect the value of PSIAs through their response to changes in relative returns if shares are to be reliably fair valued. Furthermore, the variation in PSIA values between banks is driven first by differences in the return on IAH assets, then, in decreasing order of importance, the bank's (weighted-average) cost of capital, investment account profit-sharing ratios, investment account decay rates, and finally, investment account growth rates. These findings inform bank policy – they imply that preventing account withdrawal by smoothing returns creates less value than originating high-quality assets in the first instance. This conclusion is also corroborated by our analysis of subsidies policies, wherein we show that subsidies policies – if used alone to smooth returns without the application of reserves – destroy long-term shareholder value in 77% of banks in our sample. This finding corroborates the practice of using reserves in preference to subsidies to smooth returns paid to investment account holders (Archer and Karim, 2006).

The rest of the paper is arranged as follows. Section 2 reviews the literature. Section 3 formulates a model of PSIAs from which an explicit formula for PSIA valuation is derived. Section 3 also enumerates PSIA values for banks in our sample data set and determines a rank order of PSIA value-drivers. Section 4 applies our PSIA model to investigate the impact of returns' smoothing on shareholder value creation. Section 5 concludes the paper.

2. Review of the related literature

Conventional deposits create shareholder value due to economic rents generated by paying a return to depositors less than that of an equivalent-risk investment opportunity (Hutchison and Pennacchi, 1996; Jarrow and Van Deventer, 1998; Sheehan, 2013). The economic rent for conventional deposits equals a treasury rate minus the rate paid to depositors, since government-backed deposit insurance schemes essentially render conventional deposits risk-free loans to the bank.

In contrast to conventional deposits, profit-sharing investment accounts are equity contracts with no guarantee of returns. Ex-ante uncertain profits generated from underlying assets funded by investors' capital (after provisions) are divided between the bank as managing agent (*mudharib*), and investment account holders (*rab-al-mal*). Additionally, the bank originates and manages a pool of (banking book) assets in the ordinary course of its business, a portion of which is assigned to PSIAs. Consequently, bank shareholders and investors are co-invested in the same pool of "commingled" assets (with shareholders also invested in other assets, e.g. fixed assets, solely funded by the bank) (Sundararajan, 2007; Farook et al., 2012).

The impact on shareholders' wealth due to PSIAs is from both the bank's contractual share of profits (hereafter "intrinsic" value) and the returns' management schemes it uses to administer cash returns to account holders[1]. In regards to the former, the addition to shareholders' wealth is the capitalised value of a future stream of ex-ante uncertain fee income linked to asset

[1] Other more indirect effects also arise. For example, PSIAs require Islamic banks to hold more capital, which reduces the bank's financing capacity, and in turn, the generation of earnings (see Baldwin et al. 2019).

performance. This financial claim closely resembles performance-based fees earned by fund managers for exceeding a pre-defined hurdle rate (see, for example, Goetzmann et al., 2003; Hodder and Jackwerth, 2007). The fee mechanism of mutual funds acts in the same way as the leveraged call option embedded within PSIAs. The option in PSIAs confers the contractual right of shareholders to a share of profits, with no obligation to bear losses. The call option has a strike rate of zero (akin to the hurdle rate of a fund), and a leverage ratio equal to the bank's profit-sharing ratio.

The second impact on shareholders' wealth under consideration arises from returns' smoothing. In a "dual-banking" system (for example, in Bahrain and Malaysia), Islamic banks operate alongside conventional banks and compete in the same market for retail funds. Since contractual returns to account holders are uncertain, they may fall below conventional deposit rates (so-called "rate of return risk", see Aysan et al., 2018). Due to the opportunity cost of remaining invested, some account holders withdraw their funds and switch to a higher-yielding alternative (for switching between conventional banks, see Diamond, 1971; Hutchison and Pennacchi, 1996; Vives, 2001). Consequently, Islamic banks ameliorate this withdrawal risk by administering (smoothing) cash returns paid to investors so as to shadow conventional deposit rates (Sundararajan, 2007, 2008;; Chong and Liu, 2009; Toumi et al., 2019). Implicit evidence of this returns' smoothing is reported in several studies. For example, Sundararajan (2007) finds that cash returns paid to IAHs are uncorrelated with returns generated by assets which underlie PSIAs. Additionally, a positive association between cash returns paid to IAHs and conventional deposit rates has been shown for Islamic banks in Malaysia, as reported by Chong and Liu (2009).

The impact of PSIAs on shareholders' wealth, excluding related effects of smoothing, was first reported in a seminal study on the capital structure of Islamic banks by Al-Deehani et al. (1999). Based on a small sample of 12 banks, the authors find empirical evidence that an increase in investment accounts increases the market value of Islamic banks. However, an important limitation of the study is to exclude any consideration of displaced commercial risk (DCR). DCR arises if shareholders subsidise cash returns to PSIAs by foregoing part, or all, of their contractual profit share, if assets underlying PSIAs underperform (Archer and Karim, 2006). In essence, DCR is a transference of asset risk from investment account holders to shareholders conditioned on the willingness of banks to subsidise PSIA returns when required, for example, if there is no available PER or IRR to make up shortfalls in contractual returns. Archer and Karim (2006) state that whilst subsidies reduce shareholders' wealth, for shareholders and investment account holders collectively, DCR itself is net utility-enhancing. This is because DCR efficiently reallocates risk between two classes of investors, each with different wealth and risk diversification capabilities. Subsequent to Archer and Karim (2006), only a small number of studies offer some insight into the relationship between PSIAs and shareholders' wealth. Shubber and Alzafiri (2008) find a strong positive correlation between PSIA volume and bank market value, albeit using a very small sample of banks over a short period (5-years). Other studies only indirectly relate PSIAs to shareholders' wealth through their impact on bank earnings and /or bank risk-taking (see Rosly and Zaini, 2008; Kyzy et al., 2012; Hamza and Saadaoui, 2013; Alhammadi et al., 2018). To the best of our knowledge, no studies since Archer and Karim (2006) meaningfully address our research question. We aim to fill this important gap by deriving the intrinsic value of PSIAs and evaluating the impact of returns' smoothing practices on shareholders' wealth.

3. PSIA valuation

In this section, we derive an explicit formula for PSIA value. We also calculate the account values for a sample of banks and determine an empirical rank order of PSIA value-drivers.

3.1. Valuation approach

Our concern is the *intrinsic* value of the bank's interest in PSIAs. By this, we mean the bank's share, as agent (*mudharib*) for IAHs,[2] of the expected value of discounted future profits earned *before* any transfers to/from reserves or the payment of subsidies. This point is essential to our approach. We derive PSIA value as if only *contractual* returns are paid to IAHs. By valuing PSIAs in the absence of an ex-post adjustment of cash returns, the value derived excludes both the incremental impact on future account volumes, as well as any costs to the bank, of discretionary transfers to/from IAHs. This approach is also useful because we then use the intrinsic value as a basis for evaluating the financial impact of mechanisms used by Islamic banks to shadow benchmark deposit rates (Section 4).

In common with models in the conventional deposit valuation literature (Hutchison and Pennacchi, 1996; Jarrow and Van Deventer, 1998; Sheehan, 2013), we use a contingent-claims approach. In other words, we specify ex-ante uncertain state variables whose future values determine cash flows arising from PSIAs. However, in contrast to using only one state variable for valuation purposes (which is a treasury yield in the literature last cited), we use two state variables, being the return on assets (which underlie PSIAs) and a benchmark conventional deposit rate.

From amongst the deposit valuation literature, the model of Jarrow and Van Deventer (1998) is closest to our own. The authors provide an arbitrage-free valuation of deposits in a stochastic interest rate environment with a normally distributed (exogenous) deposit rate. Analytic solutions are derived in continuous time under an assumed deposit demand function. However, unlike Jarrow and Van Deventer (1998), we do not assume the existence of a (unique) risk-neutral probability

[2] This valuation ignores any other benefits or costs to the bank flowing from its provision of PSIAs, e.g. scale benefits that increase market access to originate assets.

measure to take expectations (Cox and Ross, 1976; Harrison and Kreps, 1979). This is because a risk-neutral valuation of investment accounts requires the existence of traded securities which are perfectly correlated to the risk-drivers of PSIA value (being the return on commingled assets and the conventional deposit rate). Further, it would also need to be possible, in theory at least, to use these traded securities to construct the set of financial claims arising from PSIAs (i.e. to replicate PSIAs). However, Islamic finance prohibits both the trading of receivables for a price other than nominal receivable amount, and the short-selling of securities. These prohibitions mean that even if perfectly correlated securities could be found, it would not be possible to trade them to hedge investment accounts.

To value the bank's interest in PSIAs, we instead use real-world probabilities, and a risk-adjusted discount rate (i.e. risk-free rate plus risk premium) to calculate present values. The risk premium for the PSIA discount rate should recognise the following: first, from the bank's capital structure perspective, PSIAs are an unfunded claim on the returns of assets which underlie investment accounts; second, PSIA risk to the bank derives from unexpected changes in underlying asset returns and account volumes. In the absence of literature on the appropriate discount rate to use, we do not elaborate further on this point. We turn to each bank's weighted average cost of capital as a proxy for its PSIA discount rate.[3]

3.2. Theoretical PSIA valuation

In this section, we derive a theoretical valuation of PSIAs. Detailed proofs are in Appendix B.

3.2.1. The model

We assume a positive relationship between PSIA volume and contractual returns relative to a conventional deposit rate benchmark (i.e. contractual spread). This is supported empirically in Ergeç and Arsalan (2013) and Akhtar et al. (2017) amongst others. The main source of valuation difficulty in our model is that PSIA volume changes are random as each state variable is governed by a stochastic process. To simplify, and without loss of generality, we consider there to be only one PSIA tenor and one corresponding benchmark deposit rate. We study the total PSIA balance so that new accounts implicitly replace closed accounts.

The time t *contractual* spread, S_t, is defined by

$$S_t = \theta R_t - r_t \tag{1}$$

where θ is the pre-defined (i.e. contractual) IAH profit-sharing ratio (assumed constant), R_t is the (time t) return on IAH assets, and r_t is the (time t) conventional deposit rate benchmark. The contractual spread is the excess of the contractual return, θR_t, over the deposit rate, r_t, and can take both positive and negative values.

We next define a process governing changes in investment account volume at time t, V_t. There are (at least) two possible models we can choose. In the spirit of an economic definition of demand elasticity, we could specify $dlnV_t = \varphi dS_t$, i.e. a constant (or possibly time-varying) demand elasticity, $\varphi(>0)$, equal to the proportional change in account volume per unit change in contractual spread. However, this formulation implies that account volume only changes if there is a change in the contractual spread and that if the contractual spread took a *constant* positive (or negative) value, investment account volume would remain unchanged. This is counter-intuitive since account switching is based on the opportunity cost of remaining invested (Diamond, 1971; Hutchison and Pennacchi, 1996; Vives, 2001). Instead, therefore, we stipulate that investment account volume grows whilst the contractual spread is positive, or decays whilst it is negative (i.e. a binary occupation-time model of changes in account volume), with possibly different rates of growth and decay.

For ease of exposition, we assume that if only contractual returns are paid to IAHs (i.e. if cash returns to IAHs equal contractual returns), then changes in account volume depend on how much time the contractual return, θR_t, is either above or below the benchmark deposit rate, r_t (i.e. for how long the contractual spread, S_t, is positive or negative). Then[4]

$$\frac{dV_t}{V_t} = -\gamma 1_{\{S_t<0\}}dt + \eta 1_{\{S_t\geq 0\}}dt \tag{2}$$

where $V_t := V(R_t, r_t)$ is the time t PSIA volume, γ is the PSIA decay rate, η is the PSIA growth rate, and $1_{\{X\}}$ is the indicator function (zero if condition X is false; 1 if condition X is true).

Eq. (2) is a continuous-time[5] *occupation-time* model. It states that the proportional change in PSIA volume per unit time is a constant $-\gamma$, or η, depending on whether the contractual spread is negative or positive respectively. In this (binary) model, the absolute size of the contractual spread does not influence the size of the proportional change in PSIA volume, only whether the contractual spread is positive or negative. It is also noteworthy that whilst the growth rate (η) and decay rate (γ) depict volume

[3] A discussion of WACC for Islamic banks is provided by Al-Deehani et al. (1999). The authors state that PSIAs are equity accounts and do not impact the financial risk to shareholders. However, this treatment ignores displaced commercial risk, and trivializes the complexity of PSIAs, which are, in effect, a hybrid of debt and equity due to returns' smoothing (see Baldwin et al., 2019). A more accurate treatment of WACC for Islamic banks which encapsulates the debt-equity hybrid characteristic of PSIAs awaits further research.

[4] We could further stipulate an accelerating, instead of constant, growth rate and decay rate for values of the contractual spread progressively further away from zero, i.e. $\frac{dV_t}{V_t} = \gamma 1_{\{S_t<0\}}S_t dt + \eta 1_{\{S_t\geq 0\}}S_t dt$. However, whilst solvable, this model formulation provides little additional insight to compensate for its extra complexity. A solution for the value of PSIAs using this formulation is available upon request.

[5] A continuous time model is chosen instead of a discrete-time model for analytic tractability.

sensitivity to contractual spread in our model, their empirical values are from volume sensitivity to *cash* spread (i.e. cash returns minus benchmark deposit rate). This is because in practice, it is cash returns which are paid to IAHs.[6]

We next specify normal distributions for our state variables, the return on assets (ROA) and the conventional deposit rate, as follows:

$$dR_t = \mu_R dt + \sigma_R dB_R \tag{3}$$

$$dr_t = \mu_r dt + \sigma_r dB_r \tag{4}$$

where μ_R is the ROA drift rate, σ_R is the standard deviation of changes in ROA, B_R is a Brownian motion for shocks to ROA, μ_r is the conventional deposit rate drift rate, σ_r is the standard deviation of changes in the deposit rate, and B_r is a Brownian motion for shocks to the conventional deposit rate. B_R and B_r are correlated Brownian motions for which $dB_R dB_r = \rho dt$. Combining (1), (3), and (4), changes in the contractual spread are given by

$$dS_t = (\theta\mu_R - \mu_r)dt + \theta\sigma_R dB_R - \sigma_r dB_r \tag{5}$$

Our explanation of the model now requires a note on the assumption of normal distributions for our state variables, being the return on commingled assets, R_t, and the conventional deposit rate, r_t. The return on assets is indeed a variable for which negative values are possible (as may arise, for example, due to obligor defaults). That the conventional deposit rate changes continuously[7] and can take negative values is less defensible if considered in isolation of the overall model (a negative deposit rate may be thought of as a custodianship fee for safeguarding depositors' funds). However, it is the contractual spread (as defined in (1)) which is of primary relevance to our PSIA model, and not the return on assets or conventional deposit rate in isolation of each other. The contractual spread may indeed take both positive and negative values depending on whether contractual returns are (resp.) above or below the conventional deposit rate benchmark[8]. Lastly, assuming normally distributed state variables allows us to invoke the Feynman-Kac Theorem (Janson and Tysk (2006)) from which we derive an explicit formula for PSIA value.

We now state the value to the bank,[9] denoted Λ, of an investment (by IAHs) in a profit-sharing investment account. Since the bank shares income, but not losses,[10] in perpetuity[11]

$$\Lambda = (1-\theta)E\left[\int_0^\infty e^{-\omega t}\max(0, R_t)V_t(R_t, r_t)dt|R_0, S_0\right] \tag{6}$$

where the expectation is taken using a physical (i.e. real-world) probability measure for cash flows received by the bank from PSIAs in perpetuity, conditioned on the initial values of the return on assets (R_0) and contractual spread (S_0). ω is the continuously compounded risk-adjusted discount rate.

3.2.2. PSIA valuation model solution

Integrating (2), using $1_{\{S_t<0\}} + 1_{\{S_t\geq 0\}} = 1$, and defining $\alpha = \omega + \gamma$ and $\beta = -(\eta + \gamma)$, we restate (6) as

$$\Lambda = (1-\theta)V_0 E\left[\int_0^\infty e^{-\alpha t}\max(0, R_t)e^{-\beta\int_0^t 1_{\{S_s\geq 0\}}ds}dt|R_0, S_0\right] \tag{7}$$

where V_0 is the initial account volume.

Applying the Feynman-Kac Theorem (Janson and Tysk, 2006), the expectation in (7) is the solution to an elliptical second-order partial differential equation (PDE). Solving the PDE subject to initial values of the ROA, R_0, and the contractual spread, S_0, we derive the PSIA value per unit initial account volume

$$\Lambda = \frac{(1-\theta)R_0}{\omega-\eta}\left(1 + \frac{\eta+\gamma}{\omega+\gamma}\frac{k_1}{k_2-k_1}e^{-k_2 S_0}\right), \quad S_0 \geq 0 \tag{8}$$

$$\Lambda = \frac{(1-\theta)R_0}{\omega+\gamma}\left(1 + \frac{\eta+\gamma}{\omega-\eta}\frac{k_2}{k_2-k_1}e^{-k_1 S_0}\right), \quad S_0 < 0 \tag{9}$$

[6] The formulation of volume dynamics in Eq. (2) lends itself to a VaR measure of liquidity risk. If C is the confidence level, then the VaR level of liquidity risk, Y, over horizon T, is given by solving $C = Pr\left(ln\frac{V_T}{V_0} < Y\right)$. If the contractual spread follows a Brownian motion (as in (5)) with zero drift and initial value, it may be shown (using the mathematics of Brownian motion, see Akahori, 1995) that PSIA liquidity VaR equals $V_0\left(1 - e^{\left(\eta-(\eta+\gamma)sin\frac{\pi(1-C)}{2}\right)T}\right)$.

[7] Deposit rates are administered by banks, change intermittently, and exhibit jumps (Fan and Johansson, 2010).

[8] We applied the Jarque-Bera test to our sample data to evaluate whether the normal distribution could be used (see Section 3.3 for a description of our data sample). The null hypothesis, that the normal distribution describes the cash spread, cannot be rejected at the 1% significance level.

[9] It may be noted that valuing PSIAs as the expected value of discounted future income streams earned from the bank's funds' management role (as per Eq. (6)) is also an approach firmly embedded within the literature on the valuation of stakeholder interests in fund investments (e.g. Ferguson and Leistikow, 2001; Goetzmann et al., 2003).

[10] Income is shared continuously in our model of PSIAs. In practice, distributions of accrued income are made on a monthly basis, i.e. in discrete time.

[11] Valuing the future income stream in perpetuity is consistent with the definition of purchased goodwill, in that acquired shares are priced on a going concern basis.

where $k_1 = \frac{-\Gamma - \sqrt{\Gamma^2 + \frac{16(\omega+\gamma)}{\sigma_r^2(1-\rho^2)}}}{4} (< 0)$, $k_2 = \frac{-\Gamma + \sqrt{\Gamma^2 + \frac{16(\omega-\eta)}{\sigma_r^2(1-\rho^2)}}}{4} (> 0)$, $\Gamma = \frac{2}{\sigma_R \sigma_r \sqrt{1-\rho^2}} \left(\mu_R + (\mu_r \sigma_R - \rho \mu_R \sigma_r) \sigma_r \sqrt{1-\rho^2} \right)$, and $R_0 > 0$ (by assumption).

Proof. See Appendix B

It is noteworthy that the value to the bank of a unit investment in PSIAs given by (8) and (9) reduces to a particularly simple form for the special case in which the initial contractual spread, S_0, is zero, and for which there is no drift in ROA nor conventional deposit rate (such that the expected changes in ROA and deposit rate are both zero).

Setting $S_0 = 0$, and $\mu_R = \mu_r = 0$, the PSIA value in (8) and (9) with regularity constraint $\omega > \eta$ becomes

$$\Lambda = \frac{(1-\theta)R_0}{\sqrt{\omega - \eta}\sqrt{\omega + \gamma}} \tag{10}$$

Eq. (10) is instantly recognisable if we set $\eta = \gamma = 0$, i.e. the non-stochastic case in which the PSIA volume growth and decay rates are zero. Then, the value to the bank of a unit investment in PSIAs is $\frac{(1-\theta)R_0}{\omega}$, which is the value of a constant cash flow, $(1-\theta)R_0$, received in perpetuity discounted at rate ω. The regularity constraint $\omega > \eta$ for (10) comes from the requirement that cash flows do not grow faster than the rate at which they are discounted (otherwise the valuation is unbounded). This constraint is reminiscent of the requirement that dividends do not grow faster than the discount rate in the Gordon growth model applied to value dividend-paying equities (Gordon, 1962).

3.3. Empirical PSIA valuation

We next apply the valuation formulae (8) and (9) to determine the value of PSIAs for each bank in our sample. A majority of our data set consists of all available quarterly financial statements for a sample of 52 Islamic banks in 13 countries from Dec 2003 to Dec 2018 (being balance sheet, income statement, cash flow statement, and key financial ratios). The sample includes all Islamic retail banks in the Fitch and Eikon databases. The data set excludes Islamic windows[12] (see Appendix C for the sample country distribution). We estimate values for the following parameters: (1) investment account decay rate (γ) and growth rate (η); (2) the drift rate (μ_R) and standard deviation (σ_R) of ROA; (3) the drift rate (μ_r) and standard deviation (σ_r) for the conventional deposit rate benchmark; (4) the correlation (ρ) between the ROA of each bank and the conventional deposit rate benchmark; and (5) the weighted average cost of capital (ω) for each bank. The resulting investment account values (as a percentage of face value, or equivalently, invested capital) are presented in Fig. 1 for each bank in order of increasing value.

Fig. 1 partitions sample banks according to whether their PSIA value is less than 1% of account face value, between 1% and 20% of account face value, or above 20% of account face value. All values are in the range 0% to 66% of face value, and the average account value is 12%. The range in values we calculate for PSIAs may be compared to the range in values to conventional banks reported for savings accounts by Sheehan (2013), being from −3% to 41% of face value. Additionally, the average value of the investment accounts is within a corridor of representative values for the capitalised value of performance fees from hedge fund management reported in Goetzmann et al. (2003), being 10% to 20% of the amount invested.

One of the most striking characteristics of our results is the broad range in PSIA values of banks in our sample. This dispersion is from two factors: firstly, the numerical values for each bank of the parameters invoked by the theoretical model; secondly, the sensitivity of the account value formulae (8) and (9) to each parameter. To assess the influence of each input parameter on the dispersion of empirical PSIA values, we run two methods for the sensitivity analysis.

In the first method, we proceed as follows: (1) recalculate the value of investment accounts for each bank using the original parameter values except for one, which is the average value of the parameter across all banks in the sample; (2) repeat step (1) for each input parameter; (3) calculate the difference between the maximum account value and the minimum account value for banks in the sample for steps (1) and (2); (4) rank the input parameter according to the width of the PSIA value range (maximum value minus minimum value).

For the second method, we follow a similar procedure, except this time, the starting point is to make all of the parameter values the same for each bank (being the sample average) except for one, which, for each bank, is given its actual value. Again, we perform this step for each parameter, determining the range in investment account values, and ranking the parameters in order of their impact on dispersion.

Both methods show that the order of importance to the dispersion of PSIA values for banks in our sample is ROA first, then in decreasing order of importance, WACC, profit-sharing ratio, decay rate, and growth rate. Table 3-1 provides the width of the range in PSIA values using the first method.[13]

This finding provides insight into how Islamic banks should approach value creation through investment account management policies. Banks in our sample are more highly differentiated by return on IAH assets than profit-sharing ratios and investment account growth or decay rates. This indicates that returns' smoothing policies aimed at manipulating account

[12] Islamic windows involve conventional banks offering Islamic financial products. Such banks are regulated in accordance with rules ordinarily applied to conventional banks.

[13] Results for the second method are available upon request.

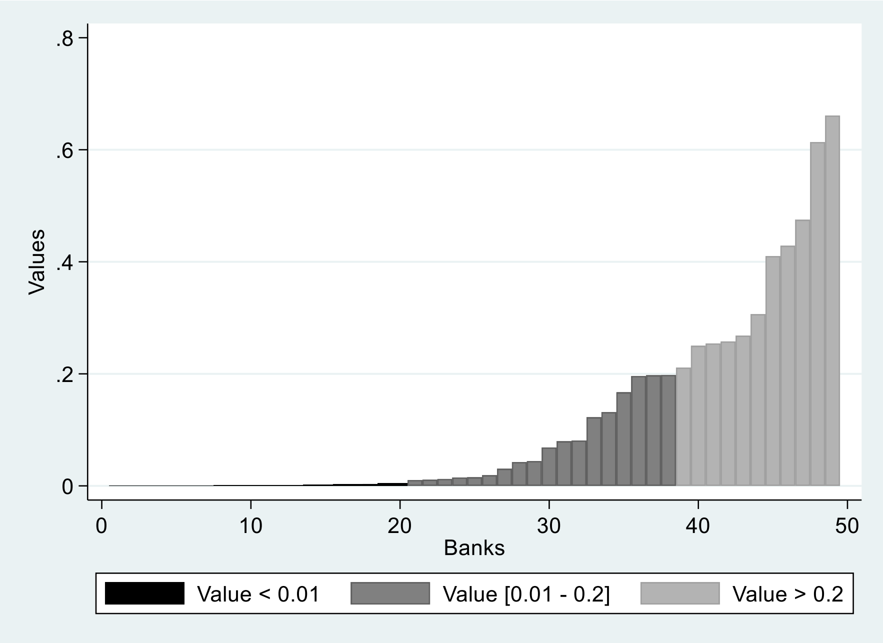

Fig. 1. The value of investment accounts for each bank.

Table 3-1
Dispersion analyses on the input parameters.

ROA	WACC	PSR	Decay Rate	Growth Rate
0.282	0.175	0.048	0.041	0.028

volumes play a lesser role in importance to shareholder value creation than those focused on the origination of high-quality assets in the first instance (i.e. those with higher risk-adjusted returns).

4. The impact of smoothing

Invoking the PSIA valuation framework developed in Section 3, we now turn to the impact of smoothing on shareholders' wealth.

4.1. Returns' smoothing schemes

Islamic banks (except in Malaysia) use two reserves to smooth returns paid to account holders. These are the Profit Equalisation Reserve (PER) and the Investment Risk Reserve (IRR) (Farook et al., 2012). The PER is an appropriation of the gross returns of IAH assets, prior to deduction of the bank's profit share, and is therefore within the equity of both shareholders and investors. Due to Shari'a restrictions, the bank cannot make up IAH losses using its own capital. Therefore, the bank's share of PER can be used to make up shortfalls in IAH returns, but not losses. IAH losses can be made whole using the IAH's share of PER, and/or the IRR, which is created by appropriations from the contractual returns due to investors after all other deductions (i.e. allocations to PER, provisions, and the bank's profit share). The IRR can also be used to make up shortfalls in IAH returns if needed (e.g. if the PER is zero).

Islamic banks may also subsidise returns paid to account holders from shareholders' equity by donating part of the bank's profit share to investors. However, the PER and IRR are intended to mitigate the need for subsidies. In other words, subsidies are only used in practice if reserves are insufficient to smooth account holders' returns (Archer and Karim, 2009).

4.2. Impact of smoothing on shareholders' wealth

The impact of smoothing on shareholders' wealth may be understood as follows. First, irrespective of the source of funds used for smoothing, if an ex-post increase in cash returns over and above the contractual profit share of investors results in more closely shadowing or exceeding the benchmark conventional deposit rate, account volume is increased. In turn, this increases the bank's future earnings, and hence, creates shareholder wealth. Second, an opportunity cost to the bank only arises in respect of smoothing when funded by bank equity that could otherwise be invested in positive NPV projects. The shareholders' portion of the PER, and all of the subsidies, are within the equity of shareholders. However, there is an important distinction between each type of appropriation of bank equity. The drawdown of PER does not result in a permanent depletion of bank equity if it is subsequently replenished by compensating asset returns (bank equity is of course initially depleted when the PER is first established). In contrast, subsidies are a donation of bank equity to investors, and are therefore a permanent appropriation of shareholders' funds without subsequent renewal. Taken in isolation of the associated benefit (of increased future earnings due to higher account volumes) accruing to shareholders, subsidies reduce shareholders' wealth. Third, the use of smoothing reserves funded by investors (the investors' portion of PER and all of the IRR) neither directly creates nor destroys shareholders' wealth. This is because these reserves are accumulated in periods when contractual returns exceed the benchmark deposit rate, and released in periods when the opposite is true. The accumulation/withdrawal of these reserves leads to an intertemporal redistribution of returns paid to account holders from within their own equity. Consequently, there is no net increase or decrease in account volume over time, and no net direct[14] impact on shareholders' wealth. As highlighted in Archer and Karim (2006), the preferential use of reserves which are within the equity of account holders to smooth returns indirectly creates wealth for shareholders only by mitigating the need to use bank equity to achieve the same objectives.

4.3. Net impact of subsidies

Because reserves have no direct net impact on shareholders' wealth (as explained in Section 4.2), our focus is the net impact of subsidies. Consider the following bank policy applied in perpetuity:

Subsidies policy

In the event of shortfalls in contractual returns, the bank reduces its profit share to pay a cash return equal to the conventional deposit rate benchmark, but pays the contractual return otherwise.

The subsidies policy is applied to make up contractual return shortfalls (but not losses) in the absence of reserves (e.g. if reserves are not permitted, as is the case in Malaysia since 2013, or available reserves are zero). The subsidies policy means that the bank provides a floor to cash returns paid to investors equal to the conventional deposit rate of return. Application of the subsidies policy permits investment accounts to grow but not to decay.

To evaluate the impact of subsidies on shareholders' wealth, we apply the PSIA model developed earlier in the paper. Under the subsidies policy, the bank pays subsidies to prevent account shrinkage in time t to $t + dt$ equal to

$$-V_t \min(S_t, 0)dt \tag{11}$$

where S_t is again the contractual spread. The subsidy in (11) is the PSIA volume, V_t, multiplied by the contractual spread, S_t (if negative, i.e. if the contractual return is below the conventional deposit rate benchmark). Payment of the subsidy means account holders receive a cash return equal to the conventional deposit rate benchmark when the contractual spread is negative, and this prevents a decrease in account volume. However, when the contractual spread is positive, no subsidy is paid. Hence, the change in account volume is given by

$$\frac{dV_t}{V_t} = \eta 1_{\{S_t \geq 0\}} dt \tag{12}$$

From (11), the lifetime cost of subsidies (i.e. the cost over all time) is

$$-E\left[\int_0^\infty e^{-\omega t} V_t min(S_t, 0)dt | R_0, S_0\right] \tag{13}$$

where V_t is given by integrating (12). The benefit of paying subsidies is to retain accounts from which the bank then receives additional earnings. The additional earnings in time t to $t + dt$ are

$$(V_t - V_0)(1 - \theta)R_t dt \tag{14}$$

where V_t is the account volume realised by implementing the subsidies policy, V_0 is the initial account volume, and $(1 - \theta)R_t dt$ is the earnings of the bank (per unit account volume) from its share of asset returns in the interval dt.

Therefore, the lifetime benefit of paying subsidies is

$$E\left[\int_0^\infty e^{-\omega t}(V_t - V_0)(1 - \theta)R_t dt | R_0, S_0\right] \tag{15}$$

Hence, a subsidies policy is value-creating for shareholders (in expectation) if

[14] An *indirect* increase in shareholders' wealth may arise from application of these reserves. For example, if the application of the reserves lowers the volatility of returns paid to investment account holders, then risk-averse account holders will benefit. This may promulgate account retention and attract new capital from which the bank earns a future share of profits.

$$E\left[\int_0^\infty e^{-\omega t}((V_t - V_0)(1-\theta)R_t + V_t min(S_t, 0))dt | R_0, S_0\right] > 0 \tag{16}$$

The left-hand side of (16) reduces to the following expressions (see Appendix D for proof):

$$\frac{(1-\theta)R_0}{\omega-\eta}\left(1+\frac{\eta}{\omega}\frac{k_3}{k_2-k_3}e^{-k_2 S_0}\right) - \frac{(1-\theta)R_0}{\omega} + \frac{e^{k_4 S_0}}{\omega(k_4-k_5)}, \quad S_0 \geq 0 \tag{17}$$

$$\frac{(1-\theta)R_0}{\omega}\left(\frac{\eta}{\omega-\eta}\frac{k_2}{k_2-k_3}e^{-k_3 S_0}\right) + \frac{e^{k_4 S_0}}{\omega(k_4-k_5)}, \quad S_0 < 0 \tag{18}$$

where $k_2 = \frac{-\Gamma+\sqrt{\Gamma^2+\frac{16(\omega-\eta)}{\sigma_r^2(1-\rho^2)}}}{4}(>0)$, $k_3 = \frac{-\Gamma-\sqrt{\Gamma^2+\frac{16\omega}{\sigma_r^2(1-\rho^2)}}}{4}(<0)$, $k_4 = \frac{-\mu_S-\sqrt{\mu_S^2+2\sigma_S^2(\omega-\eta)}}{\sigma_S^2}(<0)$, $k_5 = \frac{-\mu_S+\sqrt{\mu_S^2+2\sigma_S^2\omega}}{\sigma_S^2}(>0)$, $\Gamma = \frac{2}{\sigma_R\sigma_r\sqrt{1-\rho^2}}\left(\mu_R + (\mu_r\sigma_R - \rho\mu_R\sigma_r)\sigma_r\sqrt{1-\rho^2}\right)$, and $R_0 > 0$ (by assumption).

Using the results of our estimations for the numerical value of the parameters in (17) and (18), we find that only 12 out of 52, i.e. 23%, of banks in our sample create long-term shareholder value by following the subsidies policy.

Further insight is available with some simplifying assumptions. For zero initial contractual spread and zero drifts, (17) and (18) reduce to the requirement (see Appendix D) that

$$\frac{(1-\theta)R_0}{\sqrt{\omega-\eta}} - \frac{\sigma_S}{\sqrt{2}\eta} > 0 \tag{19}$$

In other words, the growth rate, η, must exceed a threshold value η_c, i.e.

$$\eta > \eta_c = \frac{-\sigma_S^2 + \sigma_S\sqrt{\sigma_S^2 + 8(1-\theta)^2 {R_0}^2\omega}}{4(1-\theta)^2 {R_0}^2} \tag{20}$$

This result indicates that the benefit of subsidies outweighs their costs provided accounts grow sufficiently quickly in response to increases in the cash spread.

We can also interpret the inequality in (21) from a different perspective. For a given level of account growth rate, η, the risk-adjusted return earned by the bank from PSIAs must exceed a threshold value if the subsidies policy is to result in shareholder wealth creation, i.e.

$$\frac{(1-\theta)R_0}{\sigma_S} > \sqrt{\frac{\omega-\eta}{2\eta^2}} \tag{21}$$

The left-hand side of (21) is the risk-adjusted return for the bank from its share of profits. From (21), the higher the rate of growth, η, the smaller the right-hand side becomes, and the easier this inequality is to satisfy. We also observe that accounts which provide lower risk-adjusted returns may still be value-creating, as long as they grow rapidly enough when the cash spread is positive.

5. Concluding remarks

In this paper, we derive an explicit formula for the value of profit-sharing investment accounts using probabilistic tools common to the derivatives pricing literature. We then apply the formula to value PSIAs for a sample of 52 banks in 13 countries, and determine an empirical rank order of PSIA value-drivers. We also enumerate the impact on shareholders' wealth of returns' smoothing schemes to the extent funded by shareholders' equity.

We find that PSIA values are broadly dispersed within each country and across countries. PSIA values exceed 60% of face value in some banks, whilst they are barely above zero in others. This dispersion is first attributed to differences in the return on assets underlying investment accounts, and then, in decreasing order of importance, to the bank's (weighted-average) cost of capital, profit-sharing ratio, account decay rate, and finally, account growth rate. In other words, PSIA values for banks in our sample are most differentiated by the return on assets which underlie them. The policy implication is that returns' smoothing aimed at maintaining competitive rates of return to avoid account withdrawals is less important to shareholder value creation than the origination of high-quality assets in the first instance (i.e. those with higher risk-adjusted returns).

We also find that if smoothing uses subsidies without reserves (for example if reserves are depleted or not permitted), then long-term shareholder value is destroyed in 77% of banks in our sample. This is because for these banks, the transfer of shareholder capital to make up contractual return shortfalls relative to conventional benchmark rates is not adequately compensated by a share of future profits from investment accounts subsequently retained.

Our study has several limitations. First, investors are motivated in our model by monetary returns alone, whereas religious observance may restrict switching to conventional banks. Second, conventional deposit rates change continuously in our model, whereas, in reality, deposit rates are "sticky" (i.e. change infrequently and exhibit jumps). Third, we assume a symmetric distribution to describe the return on assets which underlie investment accounts. However, assets underlying investment accounts inevitably include receivables, which have asymmetric return distributions.

By providing a model used to derive an explicit formula for PSIA value for the first time, this paper opens up several new directions for future research. One line of enquiry is the potential over- or under-payment for shares acquired in Islamic banking M&A transactions. Our model can be applied to complement industry valuation methods to increase the level of precision in calculating the fair value of equity, and in turn, determining the value of purchased goodwill.

CRediT authorship contribution statement

Kenneth Baldwin: Conceptualization, Methodology, Writing - original draft, Writing - review & editing, Visualization. **Maryam Alhalboni:** Software, Formal analysis, Investigation, Data curation, Writing - review & editing, Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Merger and acquisition activity

See Fig. A1.

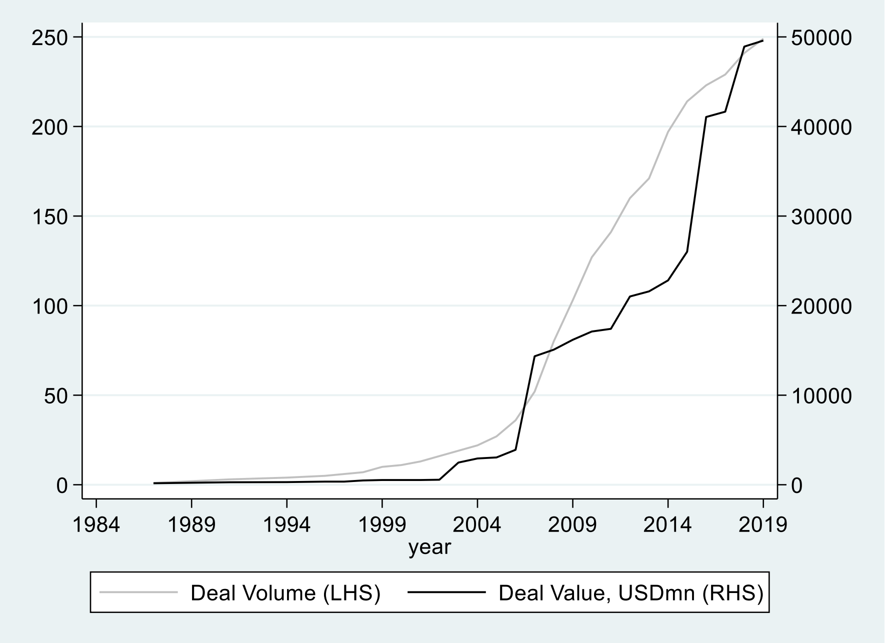

Fig. A1. Cumulative Islamic banking and financial services mergers and acquisitions 1984–2019.

Appendix B. Derivation of PSIA value

We wish to evaluate

$$E\left[\int_0^\infty e^{-\alpha t} max(0,R_t)e^{-\beta\int_0^t 1_{\{S_s\geq 0\}}ds}dt|R_0,S_0\right] \tag{B1}$$

The Feynman-Kac Theorem in 2-dimensions (Janson and Tysk, 2006) states that if $W(R_t,S_t)\geq 0$, and

$$u(t,R_0,S_0)=E\left[f(R_t,S_t)e^{-\int_0^t W(R_s,S_s)ds}|R_0,S_0\right] \tag{B2}$$

then $u(t, R_0, S_0)$ satisfies

$$\frac{\partial u}{\partial t} = Gu - Wu \tag{B3}$$

where

$$dR_t = \mu_R dt + \sigma_{RR} dB_R + \sigma_{RS} dB_S \tag{B4}$$

$$dS_t = \mu_S dt + \sigma_{SR} dB_R + \sigma_{SS} dB_S \tag{B5}$$

dB_R and dB_R are orthogonal Brownian motions, i.e.$dB_R dB_S = 0$, and the operator $G(\cdot)$ is defined by

$$G \equiv \frac{1}{2}\left(\sigma_{RR}{}^2 + \sigma_{RS}{}^2\right)\frac{\partial^2}{\partial R_0{}^2} + \left(\sigma_{RR}\sigma_{SR} + \sigma_{RS}\sigma_{SS}\right)\frac{\partial^2}{\partial R_0 \partial S_0} + \frac{1}{2}\left(\sigma_{SR}{}^2 + \sigma_{SS}{}^2\right)\frac{\partial^2}{\partial S_0{}^2} + \mu_R \frac{\partial}{\partial R_0} + \mu_S \frac{\partial}{\partial S_0} \tag{B6}$$

The derivation of PSIA value from (B1) is in 4 steps:

1. Orthogonalisation of processes for R_t and S_t.
2. Application of the Feynman-Kac Theorem
3. Transformation of the resulting PDE into its canonical form
4. Solution to the canonical form PDE

1. Orthogonalisation

We first transform the system of stochastic equations for which $dB_R dB_r = \rho dt$ and

$$dR_t = \mu_R dt + \sigma_R dB_R \tag{B7}$$

$$dS_t = (\theta\mu_R - \mu_r)dt + \theta\sigma_R dB_R - \sigma_r dB_r \tag{B8}$$

into a transformed system in which $dB_R dB_r = 0$ so that we can write

$$dR_t = \mu_R dt + X dB_R \tag{B9}$$

$$dS_t = (\theta\mu_R - \mu_r)dt + Y dB_R + Z dB_r \tag{B10}$$

where X, Y, and Z are constants which preserve the variance of dR_t, the variance of dS_t, and the covariance of dR_t and dS_t, i.e.

$$Cov(dR_t, dS_t) = \theta\sigma_R{}^2 - \rho\sigma_r\sigma_R = XY \tag{B11}$$

$$Cov(dR_t, dR_t) = \sigma_R{}^2 = X^2 \tag{B12}$$

$$Cov(dS_t, dS_t) = \theta^2\sigma_R{}^2 - 2\theta\rho\sigma_r\sigma_R + \sigma_r{}^2 = Y^2 + Z^2 \tag{B13}$$

(B11)–(B13) are solved to give

$$X = \sigma_R \tag{B14}$$

$$Y = \theta\sigma_R - \rho\sigma_r \tag{B15}$$

$$Z = \sigma_r\sqrt{(1-\rho^2)} \tag{B16}$$

i.e. we use *independent* Brownian motions dB_R and dB_r with

$$dR_t = \mu_R dt + \sigma_R dB_R \tag{B17}$$

$$dS_t = (\theta\mu_R - \mu_r)dt + (\theta\sigma_R - \rho\sigma_r)dB_R + \sigma_r\sqrt{(1-\rho^2)}dB_r \tag{B18}$$

2. Application of the Feynman-Kac Theorem

Comparing (B17) and (B18) to (B4) and (B5) gives

$$\sigma_{RR} = \sigma_R \tag{B19}$$

$$\sigma_{RS} = 0 \tag{B20}$$

$$\sigma_{SR} = \theta\sigma_R - \rho\sigma_r \tag{B21}$$

$$\sigma_{SS} = \sigma_r\sqrt{(1-\rho^2)} \tag{B22}$$

Substituting (B19) to (B22) into (B4) and (B5), invoking (B2), and specifying $f(R_t, S_t) = max(0, R_t)$ and $W(R_t, S_t) = \beta 1_{[S_t \geq 0]}$ in (B3), gives

$$\frac{\partial u}{\partial t} = \frac{1}{2}\sigma_R{}^2\frac{\partial^2 u}{\partial R_0{}^2} + (\theta\sigma_R{}^2 - \rho\sigma_R\sigma_r)\frac{\partial^2 u}{\partial R_0\partial S_0} + \frac{1}{2}(\theta^2\sigma_R{}^2 - 2\theta\rho\sigma_R\sigma_r + \sigma_r{}^2)\frac{\partial^2 u}{\partial R_0{}^2} + \mu_R\frac{\partial u}{\partial R_0} + (\theta\mu_R - \mu_r)\frac{\partial u}{\partial S_0} - \beta 1_{[S_0\geq 0]}u \tag{B23}$$

Next, define the Laplace transform of $u(t, R_0, S_0)$ s.t.

$$g(R_0, S_0) \equiv \int_0^\infty e^{-\alpha t}u(t, R_0, S_0)dt = E\left[\int_0^\infty e^{-\alpha t}max(0, R_t)e^{-\beta\int_0^t 1_{[S_s\geq 0]}ds}dt|R_0, S_0\right] \tag{B24}$$

Taking the Laplace transform of (B23), the left-hand side (using integrating by parts) becomes

$$\int_0^\infty e^{-\alpha t}\frac{\partial u}{\partial t}dt = -u(0, R_0, S_0) + \alpha g(R_0, S_0) = -max(0, R_0) + \alpha g(R_0, S_0) \tag{B25}$$

Substituting (B25) into (B23) for $R_0 > 0$ gives the PDE for $g(R_0, S_0)$,

$$\alpha g = R_0 + \frac{1}{2}\sigma_R{}^2\frac{\partial^2 g}{\partial R_0{}^2} + (\theta\sigma_R{}^2 - \rho\sigma_R\sigma_r)\frac{\partial^2 g}{\partial R_0\partial S_0} + \frac{1}{2}(\theta^2\sigma_R{}^2 - 2\theta\rho\sigma_R\sigma_r + \sigma_r{}^2)\frac{\partial^2 g}{\partial S_0{}^2} + \mu_R\frac{\partial g}{\partial R_0} + (\theta\mu_R - \mu_r)\frac{\partial g}{\partial S_0} - \beta 1_{[S_0\geq 0]}g \tag{B26}$$

3. Reducing the PDE to its canonical form

Rearranging (B26), and simplifying differential notation, gives

$$Ag_{R_0R_0} + Bg_{R_0S_0} + Cg_{S_0S_0} + Dg_{R_0} + Eg_{S_0} + Fg = G(R_0, S_0) \tag{B27}$$

where

$$A = \frac{1}{2}\sigma_R{}^2 \tag{B28}$$

$$B = \theta\sigma_R{}^2 - \rho\sigma_R\sigma_r \tag{B29}$$

$$C = \frac{1}{2}\left(\theta^2\sigma_R{}^2 - 2\theta\rho\sigma_R\sigma_r + \sigma_r{}^2\right) \tag{B30}$$

$$D = \mu_R \tag{B31}$$

$$E = \theta\mu_R - \mu_r \tag{B32}$$

$$F = -(\alpha + \beta), if S_0 \geq 0, else - \alpha \tag{B33}$$

$$G(R_0, S_0) = -R_0 \tag{B34}$$

The discriminant of the PDE (B27) is

$$\Delta = B^2 - 4AC = -(1-\rho^2)\sigma_R{}^2\sigma_r{}^2 \tag{B35}$$

For imperfectly correlated ROA and deposit rate, i.e. $|\rho| < 1$,

$$\Delta = B^2 - 4AC < 0 \tag{B36}$$

i.e. the PDE (B27) is *elliptical*. We therefore look for the following canonical form in transformed variables p and q which eliminates the second-order cross-differential term, i.e.

$$g_{pp} + g_{qq} + D^* g_p + E^* g_q + F^* g = H^* G(p, q) \tag{B37}$$

Transformation from (B27)–(B37) follows by defining

$$p = \frac{B}{2A}R_0 - S_0 \tag{B38}$$

$$q = \frac{\sqrt{4AC - B^2}}{2A}R_0 \tag{B39}$$

Further,

$$D^* = \frac{\left(D\frac{B}{2A} - E\right)}{\frac{4AC-B^2}{4A}} = \frac{2BD - 4AE}{4AC - B^2} \tag{B40}$$

$$E^* = \frac{\left(D\frac{\sqrt{4AC-B^2}}{2A}\right)}{\frac{4AC-B^2}{4A}} = \frac{2D}{\sqrt{4AC - B^2}} \tag{B41}$$

$$F^* = \frac{4AF}{4AC - B^2} \tag{B42}$$

$$H^* = \frac{4A}{4AC - B^2} \tag{B43}$$

4. Solving the canonical form PDE

First consider $S_0 < 0$. The homogeneous version of PDE (B37) is

$$g_{pp} + g_{qq} + D^* g_p + E^* g_q + F^* g = 0 \tag{B44}$$

Substituting $g(p,q) = e^{k(p+q)}$ into (B44), where k is a real-valued constant, gives

$$2k^2 + (D^* + E^*)k + F^* = 0 \tag{B45}$$

from which

$$k_1 = \frac{-(D^*+E^*) - \sqrt{(D^*+E^*)^2 - 8F^*}}{4} \quad (<0),$$
$$k_1{}' = \frac{-(D^*+E^*) + \sqrt{(D^*+E^*)^2 - 8F^*}}{4} \quad (>0)$$

For constants L and M, the complementary function is

$$g^C(p,q) = Le^{k_1(p+q)} + Me^{k_1{}'(p+q)} \tag{B46}$$

The particular integral of (B37) is

$$g^{PI}(p,q) = \frac{\sigma_R}{\alpha\sigma_r}q \tag{B47}$$

Therefore, the general solution to (B37) is

$$g(p,q) = Le^{k_1(p+q)} + Me^{k_1{}'(p+q)} + \frac{\sigma_R}{\alpha\sigma_r}q \tag{B48}$$

i.e.

$$g(R_0,S_0) = Le^{k_1\left(\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0 - S_0\right)} + Me^{k_1{}'\left(\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0 - S_0\right)} + \frac{\sigma_R}{\alpha\sigma_r}q, \quad S_0 < 0 \tag{B49}$$

But for bounded solutions to $g(R_0,S_0)$ in (B49), given $k_1 < 0$ and $k_1{}' > 0$, we require $M = 0$. Hence,

$$g(R_0,S_0) = Le^{k_1\left(\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0 - S_0\right)} + \frac{R_0}{\alpha}, \quad S_0 < 0 \tag{B50}$$

Similarly,

$$g(R_0,S_0) = Ne^{k_2\left(\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0 - S_0\right)} + \frac{R_0}{\alpha+\beta}, \quad S_0 \geq 0 \tag{B51}$$

where $k_1 = \frac{-(D^*+E^*)-\sqrt{(D^*+E^*)^2+\frac{16\alpha}{\sigma_r^2(1-\rho^2)}}}{4}(<0)$, $k_2 = \frac{-(D^*+E^*)+\sqrt{(D^*+E^*)^2+\frac{16(\alpha+\beta)}{\sigma_r^2(1-\rho^2)}}}{4}(>0)$.

We now derive values for L and N. For continuity of $g(R_0, S_0)$ at $S_0 = 0$, we require

$$Le^{k_1\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0} + \frac{R_0}{\alpha} = Ne^{k_2\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0} + \frac{R_0}{\alpha+\beta} \tag{B52}$$

Additionally, for continuity of $\frac{\partial g(R_0,S_0)}{\partial S_0}$ at $S_0 = 0$, we require

$$-Lk_1e^{k_1\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0} = -Nk_2e^{k_2\left(\theta+\frac{\sigma_r}{\sigma_R}\right)R_0} \tag{B53}$$

After algebraic manipulation, $g(R_0, S_0)$ is given by

$$\frac{1}{\alpha+\beta}\left(1-\frac{\beta}{\alpha}\frac{k_1}{k_2-k_1}e^{-k_2S_0}\right), S_0 \geq 0 \tag{B54}$$

$$\frac{1}{\alpha}\left(1-\frac{\beta}{\alpha+\beta}\frac{k_2}{k_2-k_1}e^{-k_1S_0}\right), S_0 < 0 \tag{B55}$$

where

$$k_1 = \frac{-\Gamma-\sqrt{\Gamma^2+\frac{16\alpha}{\sigma_r^2(1-\rho^2)}}}{4}(<0), \quad k_2 = \frac{-\Gamma+\sqrt{\Gamma^2+\frac{16(\alpha+\beta)}{\sigma_r^2(1-\rho^2)}}}{4}(>0) \tag{B56}$$

$$\Gamma = \frac{2}{\sigma_R\sigma_r\sqrt{1-\rho^2}}\left(\mu_R + (\mu_r\sigma_R - \rho\mu_R\sigma_r)\sigma_r\sqrt{1-\rho^2}\right) \tag{B57}$$

$$\alpha = \omega + \gamma \tag{B58}$$

$$\beta = -(\eta + \gamma) \tag{B59}$$

Substituting (B58) and (B59) into (B54) and (B55), the result follows.

Appendix C. Sample country distribution

Country	Number of Banks
Bahrain	3
Bangladesh	8
Egypt	3
Indonesia	6
Iraq	1
Jordan	3
Kuwait	4
Oman	2
Pakistan	7
Palestine	2
Qatar	3
Saudi Arabia	5
UAE	5
Total	52

Source: Fitch and Eikon databases.

Appendix D. Lifetime value of subsidies policy

We wish to evaluate

$$E_{R_0,S_0}\left[\int_0^\infty e^{-\omega t}((V_t - V_0)(1-\theta)R_t + V_t min(S_t, 0))dt\right] \tag{D1}$$

This can be re-stated

$$I_1 - I_2 + I_3 \tag{D2}$$

where

$$I_1 = (1-\theta)E_{R_0,S_0}\left[\int_0^\infty e^{-\omega t}V_tR_tdt\right] \tag{D3}$$

$$I_2 = (1-\theta)V_0E_{R_0}\left[\int_0^\infty e^{-\omega t}R_tdt\right] \tag{D4}$$

$$I_3 = E_{S_0}\left[\int_0^\infty e^{-\omega t}V_t min(S_t,0)dt\right] \tag{D5}$$

(1) Value of $\boldsymbol{I}_1$

I_1 is the value of PSIAs which can only grow and not decay, i.e. for which $\gamma = 0$. From (8) and (9),

$$\mathrm{I}_1 = \frac{(1-\theta)V_0R_0}{\omega-\eta}\left(1+\frac{\eta}{\omega}\frac{k_3}{k_2-k_3}e^{-k_2S_0}\right), \quad S_0 \geq 0 \tag{D6}$$

$$\mathrm{I}_1 = \frac{(1-\theta)V_0R_0}{\omega}\left(1+\frac{\eta}{\omega-\eta}\frac{k_2}{k_2-k_3}e^{-k_3S_0}\right), \quad S_0 < 0 \tag{D7}$$

where $k_2 = \frac{-\Gamma+\sqrt{\Gamma^2+\frac{16(\omega-\eta)}{\sigma_r{}^2\left(1-\rho^2\right)}}}{4}(>0)$, $k_3 = \frac{-\Gamma-\sqrt{\Gamma^2+\frac{16\omega}{\sigma_r{}^2\left(1-\rho^2\right)}}}{4}(<0)$, $\Gamma = \frac{2}{\sigma_R\sigma_r\sqrt{1-\rho^2}}\left(\mu_R + (\mu_r\sigma_R - \rho\mu_R\sigma_r)\sigma_r\sqrt{1-\rho^2}\right)$, and $R_0 > 0$ (by assumption).

(2) Value of I_2

The value of I_2 also follows from the value of PSIAs ((8) and (9)). We set both the growth and decay rates equal to zero for constant volume, so that

$$\mathrm{I}_2 = \frac{(1-\theta)V_0R_0}{\omega} \tag{D8}$$

(3) Value of I_3

$$I_3 = E_{S_0}\left[\int_0^\infty e^{-\omega t}V_t min(S_t,0)dt\right] \tag{D9}$$

Using $\frac{dV_t}{V_t} = \eta 1_{\{S_t\geq 0\}}dt$, this reduces to

$$I_3 = V_0E_{S_0}\left[\int_0^\infty e^{-\omega t}min(S_t,0)e^{\int_0^t \eta 1_{\{S_s\geq 0\}}ds}dt\right] \tag{D10}$$

This expectation is conditioned directly on S_0 but not on R_0, and is solved using the 1-dimensional version of the Feynman-Kac Theorem. Define

$$z(t,S_0) = E_{S_0}\left[f(S_t)e^{-\int_0^t W(S_s)ds}\right] \tag{D11}$$

where $f(S_t) = \min(S_t,0)$, and $W(S_t) = -\eta 1_{\{S_s\geq 0\}}$
Then $z(t,S_0)$ satisfies

$$\frac{\partial z}{\partial t} = \left(\frac{1}{2}\sigma_S{}^2\frac{\partial^2}{\partial S_0{}^2} + \mu_S\frac{\partial}{\partial S_0}\right)z + \eta 1_{\{S_0\geq 0\}}z \tag{D12}$$

Next, define the Laplace transform of $z(t,S_0)$ s.t.

$$g(S_0) \equiv \int_0^\infty e^{-\omega t} z(t, S_0) dt = E_{S_0}\left[\int_0^\infty e^{-\omega t} min(S_t, 0) e^{\int_0^t \eta 1_{\{S_s \geq 0\}} ds} dt\right] \tag{D13}$$

Taking the Laplace transform of the P.D.E., the left-hand side equals

$$\int_0^\infty e^{-\omega t} \frac{\partial z}{\partial t} dt = -z(0, S_0) + \omega g(S_0) = -min(S_0, 0) + \omega g(S_0) \tag{D14}$$

Therefore,

$$-min(S_0, 0) + \omega g = \frac{1}{2}\sigma_S{}^2 \frac{\partial^2 g}{\partial S_0{}^2} + \mu_S \frac{\partial g}{\partial S_0} + \eta 1_{\{S_0 \geq 0\}} g \tag{D15}$$

i.e.

$$\frac{1}{2}\sigma_S{}^2 \frac{\partial^2 g}{\partial S_0{}^2} + \mu_S \frac{\partial g}{\partial S_0} - (\omega - \eta) g = 0, \quad S_0 \geq 0 \tag{D16}$$

$$\frac{1}{2}\sigma_S{}^2 \frac{\partial^2 g}{\partial S_0{}^2} + \mu_S \frac{\partial g}{\partial S_0} - \omega g = -S_0, \quad S_0 < 0 \tag{D17}$$

For $S_0 \geq 0$, substitute $g = Ce^{k_4 S_0}$ where $k_4 < 0$ for bounded solutions and solves $\frac{1}{2}\sigma_S{}^2 k^2 + \mu_S k - (\omega - \eta) = 0$, so that $k_4 = \frac{-\mu_S - \sqrt{\mu_S{}^2 + 2\sigma_S{}^2(\omega - \eta)}}{\sigma_S{}^2} (< 0)$ (assuming $\omega > \eta$).

For $S_0 < 0$, substitute $g_c = De^{k_5 S_0}$ where $k_5 > 0$ for bounded solutions and solves $\frac{1}{2}\sigma_S{}^2 k^2 + \mu_S k - \omega = 0$ (for the homogenous equation), so that $k_5 = \frac{-\mu_S + \sqrt{\mu_S{}^2 + 2\sigma_S{}^2 \omega}}{\sigma_S{}^2} (> 0)$. The particular integral is $g_{PI} = \frac{S_0}{\omega}$, and the general solution for $S_0 < 0$ is therefore $g = g_c + g_{PI} = De^{k_5 S_0} + \frac{S_0}{\omega}$.

The constants C and D are solved using continuity in g and $\frac{\partial g}{\partial S_0}$ at $S_0 = 0$. This requires

$$C = D \tag{D18}$$

and

$$Ck_4 = Dk_5 + \frac{1}{\omega} \tag{D19}$$

Therefore,

$$C = D = \frac{1}{\omega(k_4 - k_5)} \tag{D20}$$

Hence,

$$I_3 = \frac{V_0}{\omega(k_4 - k_5)} e^{k_4 S_0}, \quad S_0 \geq 0 \tag{D21}$$

and

$$I_3 = V_0\left(\frac{1}{\omega(k_4 - k_5)} e^{k_5 S_0} + \frac{S_0}{\omega}\right), \quad S_0 < 0 \tag{D22}$$

The lifetime value of the subsidies policy, given by $I_1 - I_2 + I_3$, is therefore

$$\frac{(1-\theta)V_0 R_0}{\omega - \eta}\left(1 + \frac{\eta}{\omega}\frac{k_3}{k_2 - k_3} e^{-k_2 S_0}\right) - \frac{(1-\theta)V_0 R_0}{\omega} + \frac{V_0}{\omega(k_4 - k_5)} e^{k_4 S_0}, \quad S_0 \geq 0 \tag{D23}$$

$$\frac{(1-\theta)V_0 R_0}{\omega}\left(\frac{\eta}{\omega - \eta}\frac{k_2}{k_2 - k_3} e^{-k_3 S_0}\right) + \frac{V_0}{\omega(k_4 - k_5)} e^{k_4 S_0}, \quad S_0 < 0 \tag{D24}$$

where $k_2 = \frac{-\Gamma + \sqrt{\Gamma^2 + \frac{16(\omega - \eta)}{\sigma_r{}^2(1-\rho^2)}}}{4} (> 0)$, $k_3 = \frac{-\Gamma - \sqrt{\Gamma^2 + \frac{16\omega}{\sigma_r{}^2(1-\rho^2)}}}{4} (< 0)$, $k_4 = \frac{-\mu_S - \sqrt{\mu_S{}^2 + 2\sigma_S{}^2(\omega - \eta)}}{\sigma_S{}^2} (< 0)$, $k_5 = \frac{-\mu_S + \sqrt{\mu_S{}^2 + 2\sigma_S{}^2 \omega}}{\sigma_S{}^2} (> 0)$, $\Gamma = \frac{2}{\sigma_R \sigma_r \sqrt{1-\rho^2}}\left(\mu_R + (\mu_r \sigma_R - \rho \mu_R \sigma_r)\sigma_r \sqrt{1-\rho^2}\right)$, and $R_0 > 0$ (by assumption).

Note that for $S_0 = 0$ and zero drift, the inequality, $I_1 - I_2 + I_3 > 0$ becomes

$$\frac{(1-\theta)R_0}{\sqrt{\omega - \eta}\sqrt{\omega}} - \frac{(1-\theta)R_0}{\omega} - \frac{\sigma_S}{\sqrt{2}\omega(\sqrt{\omega} + \sqrt{\omega - \eta})} > 0 \tag{D25}$$

$$(1-\theta)R_0\left(\frac{\sqrt{\omega}-\sqrt{\omega-\eta}}{\sqrt{\omega-\eta}}\right)-\frac{\sigma_S\left(\sqrt{\omega}-\sqrt{\omega-\eta}\right)}{\sqrt{2}\eta}>0 \tag{D26}$$

As $\sqrt{\omega}-\sqrt{\omega-\eta}>0$, the inequality reduces to

$$\frac{(1-\theta)R_0}{\sqrt{\omega-\eta}}-\frac{\sigma_S}{\sqrt{2}\eta}>0 \tag{D27}$$

Further, since $\frac{\partial}{\partial\eta}\left(\frac{(1-\theta)R_0}{\sqrt{\omega-\eta}}-\frac{\sigma_S}{\sqrt{2}\eta}\right)>0$, we can define a critical value of account growth, η_c, for which $\eta>\eta_c$ creates shareholder value, and $\eta<\eta_c$ destroys shareholder value.

$$\frac{(1-\theta)R_0}{\sqrt{\omega-\eta_c}}-\frac{\sigma_S}{\sqrt{2}\eta_c}=0 \tag{D28}$$

then

$$2(1-\theta)^2{R_0}^2{\eta_c}^2+{\sigma_S}^2\eta_c-\omega{\sigma_S}^2=0 \tag{D29}$$

$$\eta_c=\frac{-{\sigma_S}^2+\sigma_S\sqrt{{\sigma_S}^2+8(1-\theta)^2{R_0}^2\omega}}{4(1-\theta)^2{R_0}^2} \tag{D30}$$

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